

General European OMCL Network (GEON) QUALITY MANAGEMENT DOCUMENT

PA/PH/OMCL (18) 147 R1 CORR

EVALUATION OF MEASUREMENT UNCERTAINTY ANNEX 1.2

Full document title and reference	Evaluation of Measurement Uncertainty ANNEX 1: ESTIMATION OF MEASUREMENT UNCERTAINTY OF USING BOTTOM-UP APPROACH Annex 1.2: Estimation of Measurement Uncertainty for Quantitative Determination Using Spectrophotometric Method (Calibration Curve Method) <i>PA/PH/OMCL (18) 147 R1 CORR</i>
Document type	Guideline
Legislative basis	Council Directive 2001/83/EC and 2001/82/EC, as amended
Date of first adoption	19 July 2019
Date of original entry into force	1 February 2020
Date of entry into force of revised document	
Previous titles/other references / last valid version	
Custodian Organisation	The present document was elaborated by the OMCL Network / EDQM of the Council of Europe
Concerned Network	GEON

N.B. This OMCL Quality Management System document is applicable to members of the European OMCL Network only. Other laboratories might use the document on a voluntary basis. However, please note that the EDQM cannot treat any questions related to the application of the documents submitted by laboratories other than the OMCLs of the Network.

**Annex 1 to Guideline “Evaluation of Measurement Uncertainty”
PA/PH/OMCL (18) 145 (in its current version)**

Estimation of measurement uncertainty of using Bottom-up approach

Annex 1.2 Estimation of Uncertainty of measurement for quantitative determination using spectrophotometric method (calibration curve method)

Example: Determination of $[\text{NH}_4]^+$ in Water

1. Description of the method

Determination of NH_4^+ in water (a parameter listed in the specification of the purified water) is performed using a dye which forms a photometric complex with ammonium. The absorbance of the ammonium-dye complex is measured at 640-660 nm.

The concentration of ammonium ($C_{\text{NH}_4^+}$) is calculated from the measured absorbance of the sample solution, using calibration curve (absorbance vs concentration of standard solutions).

Preparation of sample solution:

20.0 mL of the sample to be analysed is diluted to 25.0 ml by the chromophore solution.

Calibration curve:

Calibration curve was constructed from the relationship between absorbance and concentration of standard solutions at 5 different concentrations (0.050 mg/L, 0.100 mg/L, 0.200 mg/L, 0.250 mg/L and 0.500 mg/L). The least squares approach was applied to check the linear relationship between calibration data pairs (x_i, y_i) .

2. Estimation of the measurement uncertainty

Approach 1

2.1 Step 1. Specification of a measurand

The output quantity ($C_{\text{NH}_4^+}$) is calculated by means of the following function [4]:

$$C_{\text{NH}_4^+} = \left[\frac{A_{\text{NH}_4} - a}{b} \right] \cdot \frac{20}{25} + \Delta C_{dc}$$

Where:

A_{NH_4} : measured absorbance of the sample solution;

a and b : intercept and slope of the calibration curve ($y = a + bx$)

$\frac{20}{25}$: dilution factor, f_d

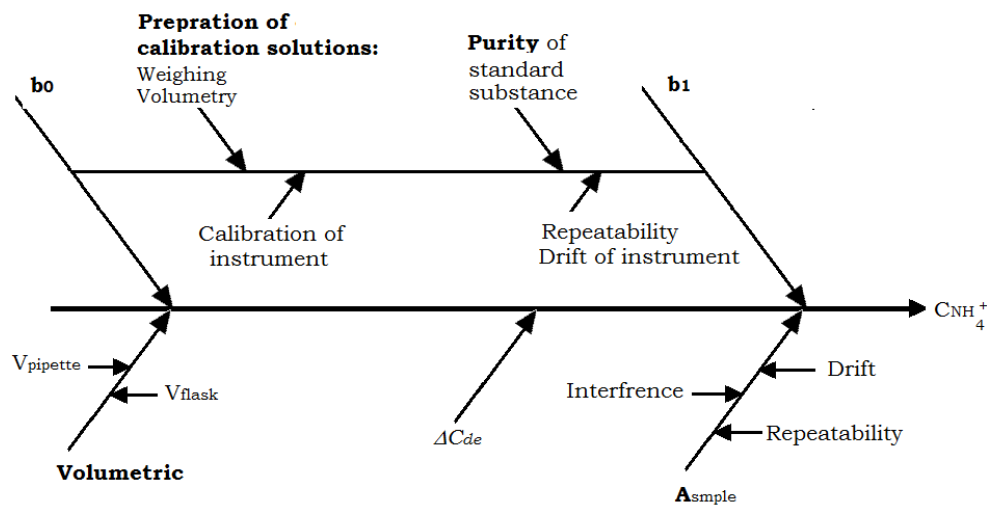
ΔC_{dc} : Component taking into account impurities

2.2 Step 2. Identification of Uncertainty Sources

Table 1. Uncertainty sources and components

Input variable	Uncertainty components	$u(x_i)$
A_{NH_4}	Interferences Repeatability Drift of photometer	$u(A_{NH_4})$
a and b	Purity of standard material Preparation of solutions Repeatability and drift of photometer	$u(a)$ $u(b)$
f_d	Preparation of sample solution	$u(f_d)$
ΔC_{dc}	Contamination Decomposition Volatilization Incomplete reaction	$u(\Delta C_{dc})$

Each of the parameters that affect the value of the measurand are shown as a cause and effect diagram (Fig. 1) [1, 2].

Fig. 1 Cause and effect diagram for determination of $C_{NH_4^+}$

2.3 Step 3. Quantification of uncertainty components

2.3.1 Component 1: Uncertainty of preparation of the sample

Uncertainty of preparation of the sample is associated with the uncertainty of the dilution factor, calculated as:

$$fd = \frac{V_2}{V_1} = \frac{25\text{mL}}{20\text{mL}} = 1.25$$

The standard uncertainty of the dilution factor $u(fd)$ is:

$$\frac{u(fd)}{fd} = \sqrt{\left(\frac{u(V_1)}{V_1}\right)^2 + \left(\frac{u(V_2)}{V_2}\right)^2}$$

The standard uncertainties $u(V_i)$ are calculated as:

$$u(V_i) = \sqrt{u(V_{cal})^2 + u(V_{rep})^2 + u(VT)^2}$$

Table 2. Uncertainty values of the dilution volume (calculation of these values is explained in Annex 1.1)

mL	Volume	$u(VT)$	$u(V_{rep})$	$u(V_{cal})$	$u(V_i)$	$\frac{u(V_i)}{V_i}$
V_1	20 mL	0.0097 mL	0.0083 mL	0.01633 mL	0.0249 mL	0.001243
V_2	25 mL	0.01212 mL	0.0125 mL	0.01225 mL	0.0273 mL	0.001093

$$\frac{u(fd)}{fd} = \sqrt{(0.001243)^2 + (0.001093)^2} = 0.00134$$

$$u(fd) = 0.00134 \cdot 1.25 = 0.001677$$

2.3.2 Component 2: Uncertainty of the absorbance of the sample solution

Uncertainty of the absorbance $u(A_{NH_4})$

Table 3. Values of the uncertainties related to $u(A_{NH_4})$

Description	Value	Unit
$u(A_{NH_4} \text{repeatability})$	0.0012	AU
$u(A_{NH_4} \text{drift})$	0.0014	AU
$u(A_{NH_4} \text{interference})$	0.0025	AU

$$u(A_{NH_4}) = \sqrt{u(A_{NH_4} \text{repeatability})^2 + u(A_{NH_4} \text{drift})^2 + u(A_{NH_4} \text{interference})^2}$$

$$u(A_{NH_4}) = \sqrt{0.0012^2 + 0.0014^2 + 0.0025^2} = 0.003106 \text{ AU}$$

Table 4. Values of the uncertainties related to absorbance of the sample solution

Description	Value	Unit
$u(A_{NH4})$	0.00311	AU
$u(\Delta C_{dc})$	0.002*	mg/mL

*The uncertainty of ΔC_{dc} is established based on experience to 0.002 mg/mL.

2.3.3 Component 3 Uncertainty of the slope and intercept of the calibration curve

There are four main sources of uncertainty to consider when estimating uncertainty of the estimated concentration x_{pred} .

1. Random variations in absorbance (both reference y_i and measured y_{obs}).
2. Random effects in assigned reference value, x_i .
3. Constant unknown offset on x_i and y_i .
4. The assumption of linearity may not be valid [3].

However, the most significant of these sources is the first source, i.e. random variations in measurement of y_i and y_{obs} , which are estimated from the residual standard deviation S [1, 4].

$$S(residual) = \sqrt{\frac{\sum_{j=1}^n [A_i - (a + b \cdot c_i)]^2}{n - 2}}$$

$$S(residual) = \sqrt{\frac{\sum_{i=0}^n [y_i - y_c]^2}{n - 2}}$$

Estimation of the uncertainties was performed by the following equations:

$$S(b) = \frac{S(residual)}{\sqrt{\sum (x_i - \bar{x})^2}}$$

$$S(a) = S(residual) \cdot \sqrt{\frac{\sum (x_i^2)}{n \sum (x_i - \bar{x})^2}}$$

Table 5. Calibration results

	x (mg/ml)	y(AU)		x_i^2	$(x-x_{mean}) = A$	$(x-x_{mean})^2$	$(y-y_{mean}) = B$	$(y-y_{mean})^2$	A*B	y_c	$(y_i-y_c)^2$
	0	0.0011									
	0.05	0.129		0.0025	-0.1700	0.02890	-0.3917	0.1534	0.0666	0.1224439	4.298E-05
	0.1	0.236		0.01	-0.1200	0.01440	-0.2847	0.0811	0.0342	0.239578	1.280E-05
	0.2	0.4696		0.04	-0.0200	0.00040	-0.0511	0.0026	0.0010	0.4738463	1.803E-05
	0.25	0.59		0.0625	0.0300	0.00090	0.0693	0.0048	0.0021	0.5909805	9.614E-07
	0.5	1.1789		0.25	0.2800	0.07840	0.6582	0.4332	0.1843	1.1766512	5.057E-06
Mean	0.2200	0.5207	Sum	0.36500		0.12300		0.67512	0.28815		0.0000798
				n=5							
	b(Slope)	2.342683			$y_c = 2.342683 x_i + 0.00531$			S(Residual)	0.0051586		
	a(Interc)	0.00531						s_b	0.014709		
	R2	0.999882						s_a	0.0039741		

Therefore, $y = 0.00531 + 2.34268x$

Using this equation y_c can be calculated with known values for x and their corresponding square of difference, i.e. $[y_i - y_c]^2$.

$$S(\text{residual}) = \sqrt{\frac{\sum_{i=0}^n [y_i - y_c]^2}{n - 2}} = \sqrt{\frac{7.9835 \cdot 10^{-5}}{3}} = 0.0051586 \text{ AU}$$

$$S(b) = \frac{S(\text{residual})}{\sqrt{\sum (x_i - \bar{x})^2}} = \frac{0.0051586}{\sqrt{0.12300}} = 0.014709 \text{ AU} \cdot \text{L/mg}$$

$$S(a) = S(\text{residual}) \cdot \sqrt{\left[\frac{\sum (x_i)^2}{n \sum (x_i - \bar{x})^2} \right]} = 0.00477 \cdot \sqrt{\frac{0.365}{5 \cdot 0.1230}} = 0.0039741 \text{ AU}$$

The standard uncertainty of a and b are approximation of the uncertainties. Because it neglects the systematic effects that impact all the points on the regression line. Furthermore, it neglects the correlation between a and b .

2.4 Step 4. Calculation of combined standard uncertainty and expanded uncertainty

The combined uncertainty is estimated by the following procedure, assuming that all variables are independent, i.e. there is no correlation between the measured values.

$$u_c = \sqrt{\left[\left(\frac{\partial C_{NH4}}{\partial A_{NH4}} \right)^2 \cdot u^2(A_{NH4}) + \left(\frac{\partial C_{NH4}}{\partial a} \right)^2 \cdot u^2(a) + \left(\frac{\partial C_{NH4}}{\partial b} \right)^2 \cdot u^2(b) + \left(\frac{\partial C_{NH4}}{\partial f_d} \right)^2 \cdot u^2(f_d) + \left(\frac{\partial C_{NH4}}{\partial \Delta Cdc} \right)^2 \cdot u^2(\Delta Cdc) \right]}$$

Replacing $\partial C_{\text{sample}} / \partial x_i$ by c_i the equation can be written as:

$$u_c = \sqrt{[c^2(A_{NH4}) \cdot u^2(A_{NH4}) + c^2(a) \cdot u^2(a) + c^2(b) \cdot u^2(b) + c^2(f_d) \cdot u^2(f_d) + u^2(\Delta Cdc) \cdot u^2(\Delta Cdc)]}$$

The partial derivatives ($\partial y / \partial x_i$) or sensitivity coefficients (c_i) describes how the output estimate (y or C_{NH4}) varies with changes in the values of the input estimates [4].

$$u(y) = |c_i| \cdot u(x_i)$$

Table 6 Estimation of the combined uncertainty by the spread sheet approach

Quantity	Value	<i>u</i>	Unit
<i>A_{NH4}</i>	0.3951	0.003106	AU
<i>a</i>	0.00531	0.003974	AU
<i>b</i>	2.342683	0.014709	AUxL/mg
<i>fd</i>	1.25	0.001677	Unitless
ΔC_{dc}	0	0.002	mg/L
<i>C_{NH4}</i>	0.207983	mg/L	

<i>u(x)</i>	0.0031064	0.0039741	0.014709	0.00167677	0.002
Quantity	<i>A_{NH4}</i>	<i>a</i>	<i>b</i>	<i>fd</i>	ΔC_{dc}
<i>A_{NH4}</i>	0.3982064	0.3951	0.3951	0.3951	0.3951
<i>a</i>	0.0053098	0.0092839	0.0053098	0.005309756	0.0053098
<i>b</i>	2.3426829	2.3426829	2.3573919	2.342682927	2.3426829
<i>fd</i>	1.25	1.25	1.25	1.25167677	1.25
ΔC_{dc}	0	0	0	0	0.002
<i>C_{NH4}</i>	0.2096	0.2059	0.2067	0.2083	0.2100
$\Delta C_{NH4} = c_i \cdot u$	0.0017	-0.0021	-0.0013	0.0003	0.0020
$(c_i \cdot u(x_i))^2$	0.000003	0.000004	0.000002	0.0000001	0.000004
Indexes	21.1%	34.6%	12.9%	0.6%	30.8%
$\Sigma [c_i \cdot u(x_i)]^2$	0.000013				
$u(C_{NH4}) = \{\Sigma [c_i \cdot u(x_i)]^2\}^{1/2}$	0.0036064				$u(C_{NH4}) = \Sigma [c_i \cdot u(x_i)]^2$
<i>U_{Expand}</i>	0.0072		<i>k</i> = 2		
(Index) ²	0.0446	0.1195	0.0168	0.0000	0.0946
<i>d_f</i>	30	4	4	30	30
(Index) ² / <i>d_f</i>	0.0015	0.0299	0.0042	0.0000	0.0032
$\Sigma (\text{Index})^2 / d_f$	0.0387				
<i>d_{f(eff)}</i>	26		<i>k</i> ≈ 2		

$$u(C_{NH4}) = \sqrt{[c_i^2 \cdot u^2(x_i)]} = \sqrt{\sum [(\Delta C_{NH4})^2]} = 0.0036 \text{ mg/mL}$$

Hence, $C_{NH4^+} = 0.208 \pm 0.0072 \text{ mg/L}$ (*k* = 2, approximately 95% level of confidence).

K = 2 is commonly used for calculation of expanded uncertainty. This value is appropriate when Type B uncertainty components dominate the uncertainty budget or when type A components have been established with ≥ 30 repeated measurements [5]. When Type A uncertainties dominate and where there are fewer than 30 degrees of freedom, a larger coverage factor is required [5]. The coverage factor in such cases is calculated by taking into account the effective degree of freedom (*df_{eff}*), which is calculated by either of the following equations [2, 5].

$$df_{eff} = \frac{1}{\sum \frac{Index_i^2}{df_i}}$$

The effective degree of freedom can also be calculated by the Welch-Satterthwaite formula [5]:

$$d_{eff} = \frac{u_c^4(y)}{\sum \frac{[c_i u(x_i)]^4}{df_i}}$$

Table 7 Calculation of the effective degree of freedom according to the Welch-Satterthwaite formula [5]

$c_i \times u(A_{NH_4})$	$c_i \times u_{(Intercept)}$	$c_i \times u_{(Slope)}$	$c_i \times u_{(df)}$	$c_i \times u_{(\Delta Cdc)}$
0.0017	-0.0021	-0.0013	0.0003	0.0020
$[c_i \times u(A_{NH_4})]^4$	$[c_i \times u_{(Intercept)}]^4$	$[c_i \times u_{(Slope)}]^4$	$[c_i \times u_{(df)}]^4$	$[c_i \times u_{(\Delta Cdc)}]^4$
0.00	2.02191E-11	2.83604E-12	1.405E-14	1.6E-11
df_i	df_i	df_i	df_i	df_i
30	4	4	30	30
$[c_i \times u(A_{NH_4})]^4/df$	$[c_i \times u_{(Intercept)}]^4/df$	$[c_i \times u_{(Slope)}]^4/df$	$[c_i \times u_{(df)}]^4/df$	$[c_i \times u_{(\Delta Cdc)}]^4/df$
2.51605E-13	5.05477E-12	7.0901E-13	4.6833E-16	5.33333E-13
Sum	6.54919E-12			
u_c	$(u_c)^4$			
0.0036	1.70212E-10		df_{eff}	26.0

$$u_c(C_{NH_4}) = \sqrt{\sum (\Delta C_{NH_4})^2} = 0.0036 \text{ mg/L}$$

Calculation of the output quantity

$$C_{NH_4^+} = \left[\frac{A_{sample} - a}{b} \right] \cdot \frac{20}{25} + C_{dc} = \left[\frac{0.3951 - 0.00531}{2.342683} \right] \cdot 1.25 + 0 = 0.208 \text{ mg/L}$$

Hence, $C_{NH_4^+} = 0.208 \pm 0.007 \text{ mg/l}$ ($k = 2$, approximately 95% level of confidence)

The contribution of each uncertainty source to the combined uncertainty is depicted in the figure 2. As seen the dilution factor (f_d) is the minor uncertainty source.

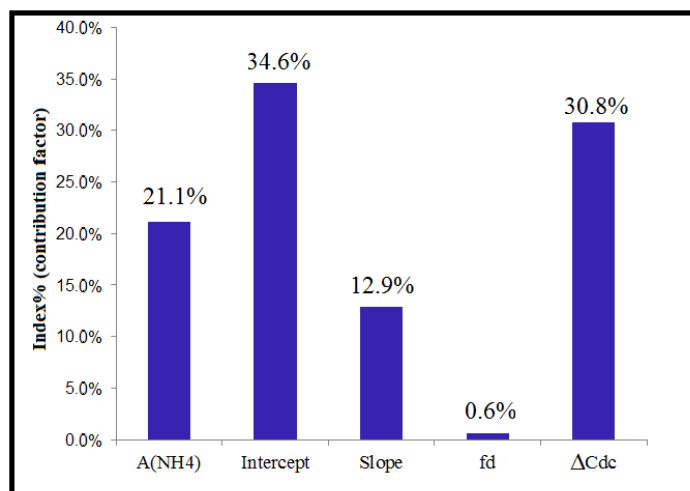


Fig. 2 Contribution of each uncertainty source to the combined uncertainty.

Approach 2

The purpose is to estimate the uncertainty for the determination of ammonium ions in water by taking into account the uncertainties in the reference standard solutions and the determined NH_4^+ concentration ($u(C_0)$ or $u(C_{x_{obs}})$) [3].

2.1 Step 1. Specification of a measurand

The output quantity ($C_{\text{NH}_4^+}$) is calculated by means of the following function [2]:

$$(C_{\text{NH}_4^+}) = c_0 \cdot fd + \Delta C_{dc}$$

Where:

$$c_0 = \left[\frac{A_{\text{NH}_4} - a}{b} \right]$$

A_{NH_4} : measured absorbance of the sample solution

a and b : intercept and slope of the calibration curve ($y = 0.00531 + 2.34268x$)

ΔC_{dc} : Component taking into account impurities

2.2 Step 2. Identification of uncertainty sources

Each of the parameters that affect the value of the measurand are shown as a cause and effect diagram (Fig. 1) [1,2].

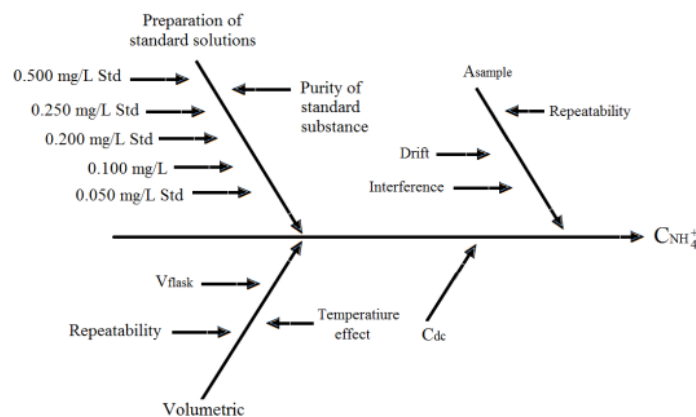


Fig. 1 Cause and effect diagram for determination of $C_{\text{NH}_4^+}$

2.3 Step 3. Quantification of uncertainty components

2.3.1 Component 1: Uncertainty of preparation of the sample

Uncertainty of preparation of the sample is associated with the uncertainty of the dilution factor, calculated as:

$$fd = \frac{V_2}{V_1} = \frac{25\text{mL}}{20\text{mL}} = 1.25$$

The standard uncertainty of the dilution factor $u(fd)$ is:

$$\frac{u(fd)}{fd} = \sqrt{\left(\frac{u(V_1)}{V_1}\right)^2 + \left(\frac{u(V_2)}{V_2}\right)^2}$$

The standard uncertainties $u(V_i)$ are calculated as:

$$u(V_i) = \sqrt{u(V_{cal})^2 + u(V_{rep})^2 + u(VT)^2}$$

Table 2. Uncertainty values of the dilution volume (calculation of these values is explained in Anex 1.1)

mL	Volume	$u(VT)$	$u(V_{rep})$	$u(V_{cal})$	$u(V_i)$	$\frac{u(V_i)}{V_i}$
V_1	20 mL	0.0097 mL	0.0083 mL	0.01633 mL	0.0207 mL	0.00104
V_2	25 mL	0.01212 mL	0.0125 mL	0.01225 mL	0.0213 mL	0.00085

$$\frac{u(fd)}{fd} = \sqrt{(0.00104)^2 + (0.00085)^2} = 0.00134$$

$$u(fd) = 0.00166 \cdot 1.25 = 0.001677$$

2.3.2 Component 2 Preparation of reference standard solutions

The reference standard solutions were prepared from a 1.000 mg/L commercial solution, according to the following scheme. As there is no information regarding the level of confidence stated for the uncertainty of the stock solution, its uncertainty was disregarded.

Table 3. Preparation scheme for standard solutions

Sample	V_{Stock} (mL)	V_{Final} (mL)	Concentration, x_i (mg/L)
Blank	-	-	0
Ref Std 1	5	100	0.05
Ref Std 2	5	50	0.10
Ref Std 3	5	25	0.20
Ref Std 4	5	20	0.25
Ref Std 5	5	10	0.50

$$C_{final} = \frac{V_{stock\ solution} \cdot C_{stock\ solution}}{V_{final}}$$

The estimated standard uncertainties $u(V_i)$ are shown in Table 4.

Table 4. Uncertainty values of the dilution volume (calculation of these values is explained in Annex 1.1)

	Volume	$u(VT)$ mL	$u(Vrep)$ mL	$u(Vcal)$ mL	$u(V_i)$ mL	$\frac{u(V_i)}{V_i}$
V_1	5 mL	0.0024	0.0010	0.01633	0.0165	0.00331
V_2	10 mL	0.00485	0.0021	0.01633	0.0172	0.00172
V_3	20 mL	0.00970	0.0083	0.01633	0.0207	0.00104
V_4	25 mL	0.1212	0.0125	0.01225	0.0213	0.00085
V_5	50 mL	0.02425	0.0128	0.03266	0.0426	0.00085
V_6	100 mL	0.04850	0.0333	0.0408	0.0716	0.00072

The uncertainties of the reference standard solutions are summarised in tables 5-9.

Table 5. Uncertainty values of the dilution volume of the first reference solution

Uncertainty component	Quantity	Uncertainty	Relative uncertainty
V_{Stock}	5 mL	0.0165 mL	0.00331
V_{Final}	100 mL	0.0716 mL	0.00072

The concentration of the first standard solution is calculated as:

$$C_{ref1} = \frac{5 \text{ mL} \cdot 1.000 \text{ mg/mL}}{100 \text{ mL}} = 0.050 \text{ mg/mL}$$

The uncertainty of the concentration of the first reference solution is:

$$\frac{u(C_{ref1})}{C_{ref1}} = \sqrt{(0.00331)^2 + (0.00072)^2} = 0.00338$$

$$u(C_{ref1}) = 0.00338 \cdot 0.05 = 0.000169 \text{ mg/mL}$$

Table 6. Uncertainty values of the dilution volume of the second reference solution

Uncertainty component	Quantity	Uncertainty	Relative uncertainty
V_{Stock}	5 mL	0.0165 mL	0.00331
V_{Final}	50 mL	0.0426 mL	0.00085

The concentration of the second standard solution is calculated as:

$$C_{ref2} = \frac{5 \text{ mL} \cdot 1.000 \text{ mg/mL}}{50 \text{ mL}} = 0.100 \text{ mg/mL}$$

The uncertainty of the concentration of the second reference solution is:

$$\frac{u(C_{ref2})}{C_{ref2}} = \sqrt{(0.00331)^2 + (0.00085)^2} = 0.00342$$

$$u(C_{ref2}) = 0.00342 \cdot 0.100 = 0.000342 \text{ mg/mL}$$

Table 7. Uncertainty values of the dilution volume of the third reference solution

Uncertainty component	Quantity	Uncertainty	Relative uncertainty
V _{Stock}	5 mL	0.0165 mL	0.00331
V _{Final}	25 mL	0.0213 mL	0.00085

The concentration of the third standard solution is calculated as:

$$C_{ref3} = \frac{5 \text{ mL} \cdot 1.000 \text{ mg/mL}}{25 \text{ mL}} = 0.200 \text{ mg/mL}$$

The uncertainty of the concentration of the third reference solution is:

$$\frac{u(C_{ref3})}{C_{ref3}} = \sqrt{(0.00331)^2 + (0.00085)^2} = 0.003416$$

$$u(C_{ref3}) = 0.003416 \cdot 0.200 = 0.000683 \text{ mg/mL}$$

Table 8. Uncertainty values of the dilution volume of the fourth reference solution

Uncertainty component	Quantity	Uncertainty	Relative uncertainty
V _{Stock}	5 mL	0.0165 mL	0.00331
V _{Final}	20 mL	0.0207 mL	0.00104

The concentration of the fourth standard solution is calculated as:

$$C_{ref4} = \frac{5 \text{ mL} \cdot 1.000 \text{ mg/mL}}{20 \text{ mL}} = 0.250 \text{ mg/mL}$$

The uncertainty of the concentration of the fourth reference solution is:

$$\frac{u(C_{ref4})}{C_{ref4}} = \sqrt{(0.00331)^2 + (0.00104)^2} = 0.003466$$

$$u(C_{ref4}) = 0.003466 \cdot 0.250 = 0.000867 \text{ mg/mL}$$

Table 9. Uncertainty values of the dilution volume of the fifth reference solution

Uncertainty component	Quantity	Uncertainty	Relative uncertainty
V _{Stock}	5 mL	0.0165 mL	0.00331
V _{Final}	10 mL	0.0172 mL	0.00172

The concentration of the fifth standard solution is calculated as:

$$C_{ref5} = \frac{5 \text{ mL} \cdot 1.000 \text{ mg/mL}}{50 \text{ mL}} = 0.500 \text{ mg/mL}$$

The uncertainty of the concentration of the fifth reference solution is:

$$\frac{u(C_{ref5})}{C_{ref5}} = \sqrt{(0.00331)^2 + (0.00172)^2} = 0.00373$$

$$u(C_{ref5}) = 0.00373 \cdot 0.500 = 0.001863 \text{ mg/mL}$$

2.3.3 Component 3. Uncertainty of the slope and intercept of the calibration curve

There are four main sources of uncertainty to consider when estimating uncertainty of the estimated concentration x_{pred} .

1. Random variations in absorbance (both reference y_i and measured y_{obs}).
2. Random effects in assigned reference value, x_i .
3. Constant unknown offset on x_i and y_i .
4. The assumption of linearity may not be valid [1].

However, the most significant of these sources is the first source, i.e., random variations in measurement of y_i and y_{obs} , which are estimated from the residual standard deviation S [1, 4].

$$S(\text{residual}) = \sqrt{\frac{\sum_{j=1}^n [A_i - (a + b \cdot c_i)]^2}{n - 2}}$$

$$S(\text{residual}) = \sqrt{\frac{\sum_{i=0}^n [y_i - y_c]^2}{n - 2}}$$

The uncertainty in x_{obs} (the determined concentration) is estimated as:

$$u(x_{obs}) = \frac{S(\text{residual})}{b}$$

Table 10. Concentration levels of reference standards and their responses (AU)

	Concentration, x_i (mg/L)	Absorbance, y_i (AU)
Blank	0	0.0010
Ref Std 1	0.05	0.1290
Ref Std 2	0.10	0.2360
Ref Std 3	0.20	0.4696
Ref Std 4	0.25	0.5900
Ref Std 5	0.50	1.1789

Table 11. Calibration results

	x (mg/ml)	y(AU)		x_i^2	$(x-x_{mean})=A$	$(x-x_{mean})^2$	$(y-y_{mean})=B$	$(y-y_{mean})^2$	A*B	* y_c	$(y_i-y_c)^2$
	0	0.0011									
	0.05	0.129		0.0025	-0.1700	0.02890	-0.3917	0.1534	0.0666	0.1224439	4.298E-05
	0.1	0.236		0.01	-0.1200	0.01440	-0.2847	0.0811	0.0342	0.239578	1.280E-05
	0.2	0.4696		0.04	-0.0200	0.00040	-0.0511	0.0026	0.0010	0.4738463	1.803E-05
	0.25	0.59		0.0625	0.0300	0.00090	0.0693	0.0048	0.0021	0.5909805	9.614E-07
	0.5	1.1789		0.25	0.2800	0.07840	0.6582	0.4332	0.1843	1.1766512	5.057E-06
Mean	0.2200	0.5207	Sum	0.36500		0.12300		0.67512	0.28815		0.000079835
				n=5							
	b(Slope)	2.342683		*$y_c = 2.342683 x_i + 0.00531$				S_(Residual)	0.0051586		
	a(Interc)	0.0053098		$c_0 (x_{obs})$	0.166386257			s_b	0.014709		
	R2	0.999882		u(x_{obs})	0.002202021			s_a	0.0039741		
				C_{NH4}	0.207982821						

Therefore, $y = 0.00531 + 2.34268x$ and

$$c_0 (x_{obs}) = (0.3951 - 0.00531) / 2.34268 = 0.166386 \text{ mg/ml}$$

Using this equation y_c can be calculated with known values for x and their corresponding square of difference, i.e., $[y_i - y_c]^2$.

$$S(\text{residual}) = \sqrt{\frac{\sum_{i=0}^n [y_i - y_c]^2}{n - 2}} = \sqrt{\frac{9.0965 \cdot 10^{-5}}{3}} = 0.0051586 \text{ AU}$$

$$u(x_{obs}) = \frac{S(\text{residual})}{b} = \frac{0.0051586}{2.342683} = 0.002220 \text{ mg/L}$$

2.3.4 Component 4: Uncertainty of the absorbance of the sample solution

Uncertainty of absorbance $u(A_{NH4})$

Table 12. Values of the uncertainties related to $u(A_{sample})$

Description	Value	Unit
$u(A_{NH4} \text{repeatability})$	0.0012	AU
$u(A_{NH4} \text{drift})$	0.0014	AU
$u(A_{NH4} \text{interfereng})$	0.0025	AU

$$u(A_{sample}) = \sqrt{u(A_{NH4} \text{repeatability})^2 + u(A_{NH4} \text{drift})^2 + u(A_{NH4} \text{interfereng})^2}$$

$$u(A_{NH4}) = \sqrt{0.0012^2 + 0.0014^2 + 0.0025^2} = 0.00311 \text{ AU}$$

Table 13. Values of the uncertainties related to absorbance of the sample solution

Description	Value	Unit
$u(A_{NH4})$	0.00311	AU
$u(\Delta C_{dc})$	0.002*	mg/mL

*The uncertainty of ΔC_{dc} is established based on experience to 0.002 mg/mL.

2.4 Step 4. Calculation of combined standard uncertainty and expanded uncertainty

Table 14. Calculated values for uncertainties

	Description	Value	Standard uncertainty $u(x)$	Relative standard uncertainty $u(x)/x$
C_{Stock}	Stock solution concentration	1.000 mg/L	-	-
C_{Ref1}	Reference standard concentration	0.050 mg/L	0.000169 mg/L	0.0033845
C_{Ref2}	Reference standard concentration	0.100 mg/L	0.000342 mg/L	0.0035501
C_{Ref3}	Reference standard concentration	0.020 mg/L	0.000683 mg/L	0.0034157
C_{Ref4}	Reference standard concentration	0.250 mg/L	0.000867 mg/L	0.0034664
C_{Ref5}	Reference standard concentration	0.500 mg/L	0.001863 mg/mL	0.0037266
x_{Obs}	Observed concentration (c_0)	0.166 mg/mL	0.0022020 mg/mL	0.0132344
A_{sample}	Absorbance of sample	0.3951 AU	0.003106 AU	0.007862
fd	Dilution factor	1.25	0.001677	0.001341

Combined standard uncertainty is calculated as:

$$\frac{u_c(C_{NH_4^+})}{(C_{NH_4^+})} = \sqrt{\left(\frac{u(C_{ref1})}{C_{ref1}}\right)^2 + \left(\frac{u(C_{ref2})}{C_{ref2}}\right)^2 + \left(\frac{u(C_{ref3})}{C_{ref3}}\right)^2 + \left(\frac{u(C_{ref4})}{C_{ref4}}\right)^2 + \left(\frac{u(C_{ref5})}{C_{ref5}}\right)^2 + \frac{u(x_{obs})}{x_{obs}} + \left(\frac{u(A_{NH_4})}{A_{NH_4}}\right)^2 + \left(\frac{u(fd)}{fd}\right)^2}$$

$$\frac{u_c(C_{NH_4^+})}{(C_{NH_4^+})} = \sqrt{(0.0033845)^2 + (0.0035501)^2 + (0.0034157)^2 + (0.0034664)^2 + (0.0037266)^2 + (0.0132344)^2 + (0.007862)^2 + (0.001341)^2} = 0.017305$$

Calculation of the ammonium concentration

The absorbance of a sample solution containing the analyte and impurities, is used for measurement of the concentration. The ammonium concentration is calculated by using the following equation:

The output quantity ($C_{NH_4^+}$) is calculated by means of the following function [2]:

$$(C_{NH_4^+}) = c_0 \cdot fd + C_{dc}, \text{ Where: } c_0 = \left[\frac{A_{sample} - a}{b} \right]$$

Table 15. Values for calculation of the $C_{NH_4^+}$

Quantity	Value	Unit
A_{sample}	0.3951	AU
a (Intercept)	0.00531	AU
b (Slope)	2.342683	AU x L/mg
R	0.9999	-
fd	1.25	-
ΔC_{dc}	0.000	mg/L
$c_0 = 0.1664$ mg/L and $C_{NH_4^+}$ in the sample = 0.208 mg/L		

$$u_c(C_{NH_4^+}) = 0.0173 \cdot 0.2084 = 0.00360 \text{ mg/L}$$

$$u_c(C_{NH_4^+}) = \sqrt{u_c(C_{NH_4^+})^2 + u_c(C_{dc})^2} = 0.00412 \text{ mg/L}$$

Expanded uncertainty ($k = 2$, approximately 95% level of confidence) is:

$$U = 2 \cdot u_c = 0.0082 \text{ mg/L}$$

2.5 Reporting of result

The result is expressed as:

Content of NH_4^+ = $0.208 \text{ mg/L} \pm 0.008 \text{ mg/L}$ ($k = 2$, approximately 95% level of confidence)

Approach 3

The purpose is to estimate uncertainty for the determined ammonium ion concentration in water by taking into account the uncertainties in the reference standard solutions and the determined NH_4^+ concentration ($u(c_0)$ or $u(x_{obs})$) [1].

2.1 Step 1. Specification of a measurand

The output quantity ($C_{\text{NH}_4^+}$) is calculated by means of the following function [2]:

$$(C_{\text{NH}_4^+}) = \left[\frac{A_{\text{sample}} - a}{b} \right] \cdot fd + C_{dc}$$

Where:

$$c_0 = \left[\frac{A_{\text{sample}} - a}{b} \right]$$

Where:

A_{sample} : measured absorbance of the sample solution;

a and b : intercept and slope of the calibration curve ($y = a + bx$)

C_{dc} : component taking into account impurities

2.2 Step 2: Identification of uncertainty sources

Table 1. Uncertainty components

Input variable	Uncertainty components	$u(x_i)$
A_{sample}	Interferences Repeatability Drift of photometer	$u(A_{\text{sample}})$
a and b	Purity of standard material Preparation of solutions Repeatability and drift of photometer	$u(a)$ $u(b)$
fd	Preparation of sample solution	$u(fd)$
C_{dc}	Contamination Decomposition Volatilization Incomplete reaction	$u(C_{dc})$

Each of the parameters that affect the value of the measurand are shown as a cause and effect diagram (Fig. 1) [1,2].

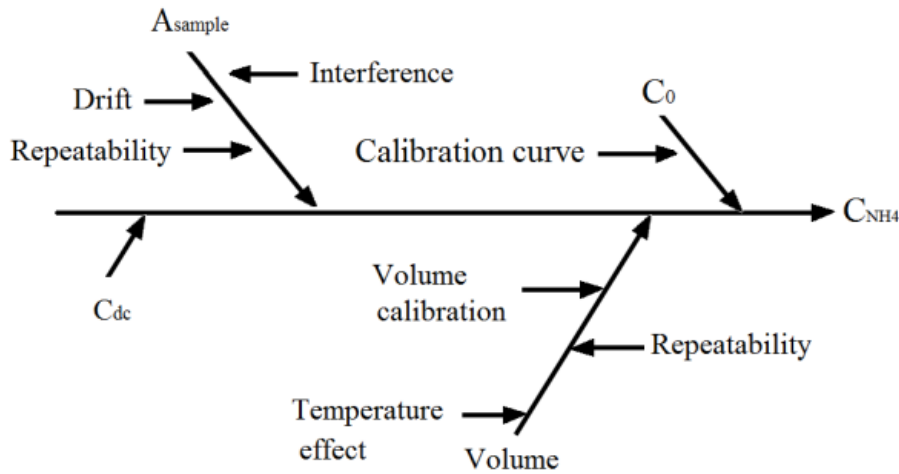


Fig. 1 Cause and effect diagram for determination of $C_{NH_4^+}$

2.3 Step 3. Quantification of uncertainty components

2.3.1 Component 1: Uncertainty of preparation of the sample

Uncertainty of preparation of the sample is associated with the uncertainty of the dilution factor, calculated as:

$$fd = \frac{V_2}{V_1} = \frac{25\text{mL}}{20\text{mL}} = 1.25$$

The standard uncertainty of the dilution factor $u(fd)$ is:

$$\frac{u(fd)}{fd} = \sqrt{\left(\frac{u(V_1)}{V_1}\right)^2 + \left(\frac{u(V_2)}{V_2}\right)^2}$$

The standard uncertainties $u(V_i)$ are calculated as:

$$u(V_i) = \sqrt{u(Vcal)^2 + u(Vrep)^2 + u(VT)^2}$$

Table 2. Uncertainty values of the dilution volume (calculation of these values is explained in Annex 1.1)

mL	Volume	$u(VT)$	$u(Vrep)$	$u(Vcal)$	$u(V_i)$	$\frac{u(V_i)}{V_i}$
V_1	20 mL	0.0097 mL	0.0083 mL	0.01633 mL	0.0207 mL	0.00104
V_2	25 mL	0.01212 mL	0.0125 mL	0.01225 mL	0.0213 mL	0.00085

$$\frac{u(fd)}{fd} = \sqrt{(0.00104)^2 + (0.00085)^2} = 0.00134$$

$$u(fd) = 0.00166 \cdot 1.25 = 0.001677$$

2.3.2 Component 2: Uncertainty of the absorbance of the sample solution

Uncertainty of absorbance $u(A_{sample})$

Table 3. Values of the uncertainties related to $u(A_{sample})$

Description	Value	Unit
$u(A_{sample\text{repeatability}})$	0.0012	AU
$u(A_{sample\text{drift}})$	0.0014	AU
$u(A_{sample\text{interfereng}})$	0.0025	AU

$$u(A_{sample}) = \sqrt{u(A_{sample\text{repeatability}})^2 + u(A_{sample\text{drift}})^2 + u(A_{sample\text{interfereng}})^2}$$

$$u(A_{sample}) = \sqrt{0.0012^2 + 0.0014^2 + 0.0025^2} = 0.00311 \text{ AU}$$

Table 4. Values of the uncertainties related to absorbance of the sample solution

Description	Value	Unit
$u(A_{sample})$	0.00311	AU
$u(C_{dc})$	0.002*	mg/mL

*The uncertainty of C_{dc} is established based on experience to 0.002 mg/mL.

2.3.3 Component 3 Calibration curve

Estimation of the uncertainty in $(u(c_0)$ or $u(x_{obs})$:

$$u(c_0) = \frac{S_{residual}}{b} \cdot \sqrt{\frac{1}{m} + \frac{1}{n} + \frac{(y_0 - \bar{y})^2}{b^2 \sum (x_i - \bar{x})^2}}$$

(it has units of mg/L)

b Slope

y-bar average value of the observed responses (= $\sum y_i/n$)

y_0 response of the sample

x_i concentration of the i:th standard solution

x-bar average value of the concentration of the standard solutions (= $\sum x_i/n$)

m number of measurements to determine C_{NH4}

n number of calibration points

Table 5. Calibration results

	x (mg/ml)	y(AU)	x_i^2	$(x-x_{mean})$	$(x-x_{mean})^2$	$(y-y_{mean})$	$(y-y_{mean})^2$	A*B	y_c	$(y_i-y_c)^2$
	0	0.0011								
	0.05	0.129	0.0025	-0.1700	0.02890	-0.3917	0.1534	0.0666	0.122443902	4.298E-05
	0.1	0.236	0.01	-0.1200	0.01440	-0.2847	0.0811	0.0342	0.239578049	1.280E-05
	0.2	0.4696	0.04	-0.0200	0.00040	-0.0511	0.0026	0.0010	0.473846341	1.803E-05
	0.25	0.59	0.0625	0.0300	0.00090	0.0693	0.0048	0.0021	0.590980488	9.614E-07
	0.5	1.1789	0.25	0.2800	0.07840	0.6582	0.4332	0.1843	1.17665122	5.057E-06
Mean	0.2200	0.5207	Sum	0.36500	0.12300		0.67512	0.28815		0.00079835
			n=5							
b(Slope)	2.342683		$y_c = 2.342683 x_i + 0.00531$				S_(Residual)	0.005159	S_{(Residual)/b}	0.00220
a(Interc)	0.0053098		c₀ (X_{obs})	0.166386			s_b	0.014709	1/m + 1/n	1.2
R2	0.999882		u (X_{obs})	0.002202			s_a	0.003974	(y₀-y_{mean})²	0.015775
			C_{NH4}	0.207983					b² · Σ(x_i-x_{mean})²	0.67504409
								u(c₀)		0.00243557

$$S(\text{residual}) = \sqrt{\frac{\sum_{i=0}^n [y_i - y_c]^2}{n - 2}} = \sqrt{\frac{7.9835 \cdot 10^{-5}}{3}} = 0.005159 \text{ AU}$$

$$u(c_0) = \frac{S_{\text{residual}}}{b} \cdot \sqrt{\frac{1}{m} + \frac{1}{n} + \frac{(y_0 - \bar{y})^2}{b^2 \sum (x_i - \bar{x})^2}} = \frac{0.005159}{2.3468} \cdot \sqrt{\frac{1}{1} + \frac{1}{5} + \frac{0.015775}{0.67504}} = 0.002436 \text{ mg/L}$$

2.4 Step 4. Calculation of Combined standard uncertainty and Expanded Uncertainty

2.4.1 Calculation of Combined standard uncertainty

$$\frac{u(c_{\text{NH}_4})}{(c_{\text{NH}_4})} = \sqrt{\left(\frac{u(A_{\text{sample}})}{A_{\text{sample}}}\right)^2 + \left(\frac{u(c_0)}{c_0}\right)^2 + \left(\frac{u(fd)}{fd}\right)^2}$$

$$\frac{u(c_{\text{NH}_4})}{(c_{\text{NH}_4})} = \sqrt{\left(\frac{0.00311}{0.3951}\right)^2 + \left(\frac{0.002202}{0.16639}\right)^2 + \left(\frac{0.00168}{1.25}\right)^2} = 0.01545$$

2.4.2 Calculation of the output quantity

Table 16. Values for calculation of the $C_{\text{NH}_4^+}$

Quantity	Value	Unit
A _{Sample}	0.3951	AU
a (Intercept)	0.00531	AU
b (Slope)	2.34268	AU x L/mg
R	0.9999	-
fd	1.25	-
ΔC_{dc}	0.000	mg/L
$c_0 = 0.1664 \text{ mg/L}$ and $C_{\text{NH}_4^+}$ in the sample = 0.2079 mg/L		

$$u(C_{\text{NH}_4^+}) = 0.01545 \cdot 0.2079 = 0.0032125 \text{ mg/L}$$

Calculation of the output quantity

$$C_{\text{NH}_4^+} = 0.2079 \text{ mg/L}$$

Hence, $C_{\text{NH}_4^+} = 0.208 \pm 0.007 \text{ mg/L}$ ($k = 2$, approximately 95% level of confidence)

4. Summary

The results of uncertainty estimations, using different methods, are summarised in the following Table.

Table 8 Summary of the obtained results for $u_c(C_{\text{NH}_4^+})$ using different approaches

Estimation of $u_c(C_{\text{NH}_4^+})$	Determined concentration (mg/L)	Estimated $U_{\text{Expanded}} (k = 2)$ mg/mL
Approach 1	0.2084	0.007
Approach 2	0.2084	0.008
Approach 3	0.2084	0.007

5. References

1. Eurachem/CITAC Guide CG 4, Quantifying Uncertainty in Analytical Measurement, Third Edition, 2012.
2. Estimation of measurement uncertainty in chemical analysis (analytical chemistry) course. An introductory course on measurement uncertainty estimation, specifically related to chemical analysis. Ivo Leito, Lauri Jalukse, Irja Helm.
3. A Guide on Measurement Uncertainty in Chemical & Microbiological Analysis. Accreditation Scheme for Laboratories, Technical Guide 2. Second edition 2008.
4. Statistics and Chemometrics for Analytical Chemistry, JC Miller and JN Miller, 6th edition, Ellis Horwood Prentice Hall, 2010.
5. G104- Guide for Estimation of Measurement Uncertainty In Testing, American Association for Laboratory Accreditation, December 2014.